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Classifier of scientific area: UDK 519.862.6 - Econometrics.

Problem – Modeling and analysis of risk in complex systems.

Essence – We introduce groups of incompatible events or countable sets to the data base to get the system of logical equations and knowledge base and to use LP-calculus.

SCENARIO LOGIC AND PROBABILISTIC BRIBE RISK MODELS

True logic of our world
is calculation of probabilities.
Maxwell, J. C.

Summary. The scenario logic and probabilistic (LP) bribe risk models for departments “Economic crimes” of towns and services of safety and control of banks and companies are offered with the purpose of revealing, estimating and analyzing bribes on the basis of the statistical data. The following bribe risk LB-models are described: 1) at the institutions according to the results of their functioning, 2) of the officials on the basis of the descriptions of their behavior, 3) of the institution and of the officials on the basis the analysis of the service parameters. Problems of bribes and corruption are of great computing complexity and are solved only by means of special Software.

Key words: *bribe, corruption, logic, risk, probability, theory, events, scenario, model, estimation, analysis, statistics*

Problems of bribes and corruption have been actual at all times and in all countries. Now there are a lot of articles on www.vzyatka.ru about the flourishing of bribes and about the corruption. Books and articles on corruption and on bribes [1, 2], on social statistics [3, 4] have thorough substantial descriptions and analysis, as well as a great number of various examples, comments on the law and on the criminal codex, but they do not contain any mathematical models of bribes.

The adequate mathematical apparatus is needed for the solution of social and organizational problems, including problems of revealing various frauds, bribes and corruption. It must be based, according to John von Neumann and Norbert Wiener [5], on logic, on discrete mathematics and on combination theory.

Such adequate mathematical apparatus is being developed and it is called “Logic and probabilistic (LP) theory of risk with the groups of incompatible events” [5, 7]. It has been tested for the estimation and for the analysis of credit risks, security portfolio risk, the risk of the loss of quality, the risk of non-success of the management of the company. The LP-models of risk have a high quality. For example, the credit LP-models of risk have shown the accuracy almost two times higher, the robustness almost seven times greater than other methods and also an absolute transparency in classifying than other methods.

In the present work an attempt has been made to use the LP-approach and the LP-calculus [5-7] for the solution of the actual problem – for the estimation and analysis of the probability of bribes and corruption.

1. Axioms of the bribe risk theory

Corruption is regarded as the basic kind of the so called shadow economy. More often corruption implies the reception of bribes and illegal monetary incomes by state bureaucrats who extort them from citizens for the sake of personal enrichment. That is a brazen violation of public morals and of the norms of law.

For the construction of the system and the technique of the struggle against bribes and corruption the following axioms have been accepted [5, 7]:

- Under the pressure of circumstances everyone may swindle if valuables are not guarded well enough and if it is possible to conceal the trickery for some time and when the control over the validity of the decisions taken is insufficient.
- Without a quantitative estimation and without the analysis of the probability of bribes it is impossible to struggle against the swindle, bribes and corruption.
- Each commercial bank or company is capable of a swindle or corruption if there is no transparency in their business and no control over their activities.
- Behind the non-transparency of the techniques of the estimation of credit risks and ratings of the banks and of the borrowers there may be bribes and swindles.
- Complexity of the organizational structure of an institution or company can be a sign of swindle and corruption.

Concepts of the probability of bribes and corruption are close to those of reliability and safety in engineering and they are also close to the notion of risk in economy, in business and in banks. Most frequently bribes take place when people receive licenses (in education, tourism, medicine, construction), sanctions (GAI, customs), in education (certificates, diplomas, examinations), registration (bodies of the Ministry of Internal Affairs, embassy, bodies of local authorities), and etc.

The scenarios and the technique of a bribe are various for the ministry, for the mayoralty, for institutions, for companies, for banks, for officials, for doctors, for teachers, etc. The Bribe implies two objects: the briber and the bribe-taker, either of whom has his benefit. The briber solves his problem faster, more qualitatively, receives privileges, bypasses the law, etc. The bribe-taker has monetary or material benefit, etc. We use the following terms: probability of corruption and of a bribe, probability of success and of non-success, probability of the absence or of the presence of a bribe, probability of a good or bad project (of an object, of an official, of an institution) We consider those terms from the point of view of the size of the probability of a bribe.

For a quantitative estimation and for the analysis of the bribe probability we use the logical and probabilistic non-success risk LP-theory (LP-theory) with groups of incompatible events (GIE) [5, 7-9], and some bribe LP-models are constructed on the basis of the statistical data. The paper is one of the first mathematical publications on the probability of bribes and does not claim to consider all the aspects of this complex problem or to develop all the scenarios of bribes. Here we just give the description and the construction of the bribe model, try to give the estimation and the analysis of the probability of a bribe, and hardly ever touch upon the social, legal and organizational problems of bribes.

2. The risk LP-theory of a bribe with groups of incompatible events

Events and probabilities. An event of a bribe is described by signs and their grades, which happen to be random variables and are regarded as the logic variable of casual sign-events and grade-events having certain probabilities. The sign-events are connected by the logic connections *OR*, *AND*, *NOT* and can have cycles. Grade-events for a sign make a group of incompatible events (GIE) [5, 7].

Signs are the characteristics of an object (of a process, of a project) for which special measurement scale–grades are used: the logical scale (the truth \ the lies, $1 \setminus 0$), the qualitative scale (high, average and low salary), the numerical scale (intervals $[a, b]$, $[b, c]$), etc. Generally, the grades

are linearly disordered and it is impossible to tell whether grade 3 is worse or better for the final event than grade 4.

The logical variable Z_j corresponding to a sign-event, is equal to 0 with the probability P_j if the sign j testifies that a bribe has taken place there, and it is equal to 1 with the probability $Q_j=1-P_j$ in case of the absence of a bribe. The logical variable Z_{jr} corresponding to the grade r for the sign j , is equal to 0 with the probability P_{jr} and is equal to 1 with the probability $Q_{jr}=1-P_{jr}$. The vector $Z(i)=(Z_1, \dots, Z_j, \dots, Z_n)$ describes the object i on the basis of statistics. When the object i is given instead of the logical variables $Z_1, \dots, Z_j, \dots, Z_n$, it is necessary to substitute the logical variables Z_{jr} for the grade of the sign of this object.

The bribe risk L-function in a general way is as follows

$$(1) \quad Y = Y(Z_1, Z_2, \dots, Z_n).$$

The bribe risk P-function in a general way is

$$(2) \quad P_i\{Y = 1 | Z(i)\} = P(P_1, \dots, P_j, \dots, P_n), i = 1, 2, \dots, N.$$

Three probabilities have been considered for every grade-events in GIE (Fig. 1): $P2_{jr}$ is a relative frequency in the statistics; $P1_{jr}$ is a probability in GIE; P_{jr} is the probability, substituted into (2) instead of the probability P_j . We will determine these probabilities for the j -th GIE:

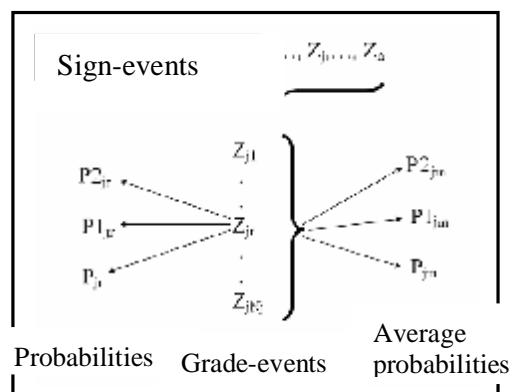


Fig.1. Probabilities in the groups of incompatible events

$$(3) \quad P2_{jr} = P\{Z_{jr} = 1\}; \sum_{r=1}^{Nj} P2_{jr} = 1; r = 1, 2, \dots, Nj;$$

$$(4) \quad P1_{jr} = P\{Z_{jr} = 1 | Z_j = 1\}; \sum_{r=1}^{Nj} P1_{jr} = 1; r = 1, 2, \dots, Nj;$$

$$(5) \quad P_{jr} = P\{Z_j = 1 | Z_{jr} = 1\}; r = 1, 2, \dots, Nj, j = 1, 2, \dots, n.,$$

where n is the number of signs; N_j is the number of grades in the j -sign, the vertical hyphen (|) is read provided.

The average values of the probabilities $P2_{jr}$, $P1_{jr}$ and P_{jr} for the grades in GIE are equal to:

$$(6) \quad P2_{jm} = 1 / N_j; P_{jm} = \sum_{r=1}^{Nj} P_{jr} P2_{jr}; P1_{jm} = \sum_{r=1}^{Nj} P1_{jr} P2_{jr}.$$

We will estimate the probabilities P_{jr} during the algorithmic iterative identification of the P-model of the bribe on the basis of the statistical data. In the beginning it is necessary to determine the probabilities $P1_{jr}$ satisfying (4) and to pass from the probabilities $P1_{jr}$ to the probabilities P_{jr} . The number of the independent probabilities is equal to:

$$(7) \quad N_{ind} = \sum_{j=1}^n N_j - n.$$

The probabilities P_{jr} and PI_{jr} are connected with the Bayes formula for the case of a limited quantity of information by way of the average probabilities P_{jm} and PI_{jm} [5]:

$$(8) \quad P_{jr} = PI_{jr} * (P_{jm} / PI_{jm}); r = 1, 2, \dots, N_j; j = 1, 2, \dots, n.$$

Identifying the bribe risk LP-model on the basis of statistical data. The problem of the identification of the bribe risk LP-model is solved by the algorithmic methods [5, 7]. The following scheme of solving the problem is proposed here. Let the probabilities for the grades $P_{jr}, r=1,2,\dots,N_j; j=1,2,\dots,n$ be known as a first approximation; and the risks $P_i, i=1,\dots,N$ for the objects in statistics be also calculated, each of which might be accompanied by bribes. In the statistics of good projects the symbol N_g is used and in the statistics of bad projects the symbol N_b is used. We will determine the admitted risk P_{ad} (Fig. 2) so that the number of projects accepted by us without bribes (the good projects) N_{gc} had the risk lesser than the admitted one and, accordingly, the number of the projects with bribes (the bad projects) $N_{bc}=N - N_{gc}$ had the risk which exceeded the admitted one. At the step of optimization we shall change the probabilities $P_{jr}, r=1,2,\dots,N_j; j=1,2, \dots,n$ in such a way that the number of the recognizable projects might be increased. The variables P_{ad} and N_{gc} are connected unequivocally. In the algorithm of the problem it is more convenient to use N_{gc} and to determine the admitted risk P_{ad} .

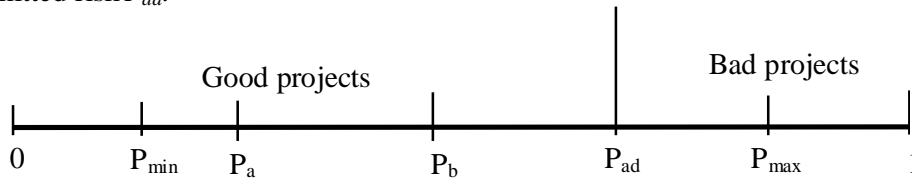


Fig. 2. The scheme of the classification of projects

The condition $P_i > P_{ad}$ specifies the following types of projects: N_{gg} – denotes the projects which are good according to both - their methods and the statistics; N_{gb} – denotes the projects which are good according to their methods and bad by the statistics; N_{bg} – denoted the projects which are bad by their methods and good by the statistics; N_{bb} – denotes the projects bad by both the methods and the statistics. The risks of the projects $N_{gg}, N_{bb}, N_{gb}, N_{bg}$ move relative to P_{ad} at the change of P_{jr} . At the transition of some projects which are bad to the right from P_{ad} on value of the risk, the some number of projects passes to the left. The change P_{jr} which translates the projects N_{gb} and N_{bg} through P_{ad} towards each other will be the optimal one.

The problem of the identification of the P-model of the bribe is formulated like this [5, 7]. The statistics on the bribes, having N_g good and N_b bad projects and the bribe P-model (2) *have been assigned. It is required to determine the probabilities $P_{jr}, r=1,2,\dots,N_j; j=1,2,\dots,n$ grade-events and the admitted risk P_{ad} , dividing the projects into the good and bad ones. The goal function is as follows: the number of the projects which are to be correctly classified should be maximal*

$$(9) \quad F = N_{gg} + N_{bb} \rightarrow \underset{P_{jr}}{\text{MAX}},$$

From expression (9) it follows that the accuracy of the P-model of the bribe in the classification of the good objects E_g , the bad objects E_b and as a whole E_m is equal to

$$(10) \quad E_g = N_{gb} / N_g; E_b = N_{bg} / N_b; E_m = (N - F) / N.$$

Restrictions:

$$(11) \quad 1) \text{ The probabilities } P_{jr} \text{ should satisfy the condition:} \\ 0 < P_{jr} < 1, j = 1, 2, \dots, n; r = 1, 2, \dots, N_j.$$

2) The average risks of the projects on the P-model and according to statistics should be equal; at identifying the P-model of risk we shall correct the probabilities P_{jr} on a step by the formula

$$(12) \quad P_{jr} = P_{jr}(P_{av} / P_m); j = 1, 2, \dots, n; r = 1, 2, \dots, N_j,$$

where: $P_{av} = N_b / N$ is the average risk according to statistics; P_m is the average risk on the model.

3) The admitted risk P_{ad} should be determined at the given factor of asymmetry of the recognition of the good and bad projects, equal to

$$(13) \quad E_{gb} = N_{gb} / N_{bg}.$$

The formula for identifying the bribe risk LP-model

$$(14) \quad \Delta P_{jr} = K_1 \frac{N_{opt} - N_v}{N_{opt}} K_3 P_{jr}, j = 1, 2, \dots, n; r = 1, 2, \dots, N_j,$$

where: K_1 is the factor equal approximately to 0,05; N_{opt} , N_v are the number of optimization and the number of the current optimization respectively, K_3 is a random number in the interval $[-1, +1]$. During the optimization the size ΔP_{jr} tends to zero.

The analysis of bribe probability. Let the bribe risk P-model and the probabilities P_{jr} be defined and known. We shall determine the contributions of sign-events and grade-events into the probability of the bribe for the project and for a set of projects, and also into the accuracy of the bribe LP-model. For this purpose we shall calculate the differences between the values of the characteristics for the optimal model and if the probabilities are endowed with the corresponding grade-events of zero values [5, 7].

The contribution of a sign (of all the grades of a sign) into the probability of a bribe for the project i

$$(15) \quad \Delta P_j = P(i) - P(i) \Big|_{P_j=0}, j = 1, 2, \dots, n.$$

The contribution of an attribute into the average probability of the bribe P_m of a set of projects

$$(16) \quad \Delta P_{jm} = P_{jm} - P_{jm} \Big|_{P_j=0}, j = 1, 2, \dots, n.$$

The contribution of an attribute into the goal function F_{max}

$$(17) \quad \Delta F_j = F_{max} - F \Big|_{P_j=0}, j = 1, 2, \dots, n.$$

Contributions of grades into the goal function ΔF_{jr} will be calculated by us by analogy with (10) as mistakes of the classification of projects on every grad-event:

$$(18) \quad E_{jrg} = N_{jr gb} / N_{jrg}; E_{jrb} = N_{jr bg} / N_{jrb}; \\ E_{jrm} = (N_{jr gb} + N_{jr bg}) / N_{jr},$$

where $N_{jr gb}$, $N_{jr bg}$, N_{jr} are the numbers of the non-correct good and bad project and of all the projects with the grade r of the sign j .

By the results of the analysis of the contributions into the probability of the bribe grades, of the signs, of the project or a set of projects, the bribe model for the increase of its accuracy is optimized.

3. The bribe risk LP-model at an institution

The institution making decision on some projects (on the cases or affairs of the citizens). There are a lot of projects. The projects are either successful (good) or non-successful (bad). The reasons for the non-success of the projects are the unjustified sanctions, given out as a result of bribes.

Elements of the scenario and of the bribe risk LP-model are the functional departments $Z_1, \dots, Z_j, \dots, Z_n$, each of which has N_j officials who make decisions.

Generally the object with the elements $Z_1, \dots, Z_j, \dots, Z_n$ is complex as it includes connections *OR*, *AND*, *NOT*, and repeated elements and cycles. Officials in the j -department $Z_{j1}, \dots, Z_{jr}, \dots, Z_{jN_j}$ are GIE. The official, making a decision, signs the corresponding document. The construction of the

bribe LP-model consists in the calculation of the probabilities $P_{jr}, j=1,2,\dots,n; r=1,2,\dots, N_j$ with which officials take bribes on the basis of the statistics from N successful and non-successful projects.

We shall consider the bribe LP-model, say, of a bank. The statistics about the success of the credits is used. The reasons of the non-success of the credits are explained by bribes.

Let the bank has five functional groups of the officials who take decisions on giving out the credits. The logic variables Z_1, Z_2, Z_3, Z_4, Z_5 correspond to these functional groups. These groups have accordingly N_1, N_2, N_3, N_4, N_5 officials taking decisions. The number of the officials in groups coincides with the number of the grades in GIE.

The given credits are either successful (grade 1) or non-successful (grade 0). There are documents on the given credits where the officials making decision fix their signatures.

The greatest number of any possible combinations of the client's passing through the institution and the bribes is equal to

$$(19) \quad N_{\max} = N_1 g N_2 g N_3 g N_4 g N_5.$$

The logic function of the bribes in the perfect disjunctive normal form (PDFN) has N_{\max} of the logical terms and we may write

$$(20) \quad Y = Z_1 Z_2 Z_3 Z_4 Z_5 \vee \overline{Z_1} Z_2 Z_3 Z_4 Z_5 \vee Z_1 \overline{Z_2} Z_3 Z_4 Z_5 \vee \dots \vee \overline{Z_1} \overline{Z_2} \overline{Z_3} \overline{Z_4} \overline{Z_5}.$$

Every logic variable from Z_1, Z_2, Z_3, Z_4, Z_5 or its denial (the line over a variable) goes into any conjunct. All the conjuncts are orthogonal in pairs, that is PDFN is the orthogonal form of the logic function. At calculation of the probability of the event Y we put in (20) the probabilities P_1, P_2, P_3, P_4, P_5 instead of the events Z_1, Z_2, Z_3, Z_4, Z_5 and the sign "OR" replace by "+".

PDFN is the cumbersome recording the logic function. In really the logic bribe model may be recorded simpler if taking into account of the structure of bank departments and their connection. It can be of any kind. To be specific, we will assume that the structure of the risk model is presented by the "bridge" (Fig. 3).

The officials from Z_1 and Z_2 check the maintenance of the credits, and the officials from Z_3 and Z_4 take the decision on the size and on the terms of the credit. The top officials (chiefs) from Z_5 control the process. The client visits one of the top officials who either advises the client or takes a bribe and directs the client to the officials from the groups Z_3 or Z_4 who take bribes too.

The logic model (L-model) of bribes in disjunctive normal form (DNF) (the records of the logic functions without the brackets) on the basis of the shortest way of functioning is

$$(21) \quad Y = Z_1 Z_3 \vee Z_2 Z_4 \vee Z_1 Z_5 Z_4 \vee Z_2 Z_5 Z_3.$$

The probabilistic model (P-model, P-polynomial) of bribes, obtained after the orthogonalisation of the logic function (19)

$$(22) \quad P = p_2 p_4 + p_1 p_3 + q_1 p_2 p_3 q_4 p_5 + p_1 q_2 q_3 p_4 p_5 - p_1 p_2 p_3 p_4.$$

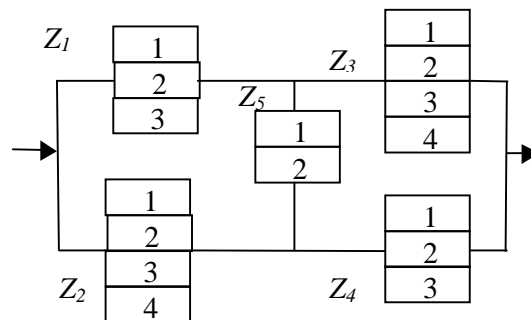


Fig. 3. Structural model of the risk bribes of the "bridge" type

Example 1. For training the bribe P-model the statistics from 1000 credits is used (700 are good and 300 are bad). The average bribe risk is equal to $P_{av}=300/1000=0,3$. Five sign-events have from 4 to 11 grades; all in all there are 40 grades.

As a result of training, the probabilities P_{jr} and PI_{jr} for all grade-events have been obtained and the following parameters of the bribe LP-model have been calculated: the goal function is equal to $F_{max}=720$ and the admitted risk is equal to $P_{ad}=0,3094$.

Some results of computing researches are given in Tables 1 and 2. The probabilities $P2_{jr}$ and PI_{jr} of the grades though they make a total 1 in GIE, can differ essentially (Table 2). The probabilities of bribes (probabilities P_{jr}) differ more than 10 times. The sign-events 1 and 4 have the maximum average probabilities P_{jm} . The same events bring the maximum contributions to the average risk P_m . The average probabilities P_{jm} for sign-events differ nearly two times.

Table 1. Average probabilities of bribes for the groups of officials

Groups, j	Probabilities, P_{jm}	Probabilities, PI_{jm}	Number of officials, N_j
1	0,478113	0,249540	4
2	0,348310	0,075949	10
3	0,298833	0,133823	5
4	0,388857	0,116348	11
5	0,291868	0,091775	10

Table 2. Probabilities of bribes og officials

Numbers of grades	Probabilities, P_{jr}	Probabilities, PI_{jr}	Frequencies, $P2_{jr}$
Group Z_1			
1	1,0	0,522300	0,274
2	0,596084	0,311103	0,269
3	0,248278	0,129579	0,063
4	0,070927	0,037017	0,394
Group Z_2			
1	0,0	0,0	0,0
2	0,687703	0,149933	0,014
3	0,227359	0,0495688	0,002
4	1,0	0,218209	0,054
5	0,510577	0,111316	0,017
6	0,704722	0,153643	0,086
7	0,570149	0,124304	0,057
8	0,448856	0,097859	0,224
9	0,434821	0,094799	0,187
10	0,001675	0,000365	0,359

4. The bribe risk LP-model on the basis of the description of the officials' behavior

A bribe is not a crime which is made a parade. There is no question about "corpus delict" at a robbery of a bank which is witnessed by the employees or by the clients. A bribe differs from any other kinds of crime by the difficulty of its revealing. However, bribes have a mass character and there are many data on bribes both in judicial law-courts, and in the controlling units.

For each type of bribes it is possible to find signs [1, 5] which are associated with a similar crime. Each of such signs has at least 2 grades. The P-model bribe can be identified on the statistical data.

The investigation of the bribe can be carried out only in the case when there are serious reasons to believe that the bribe had actually taken place in the past. The value of this "seriousness" can be estimated quantitatively on the probability of the bribe, and the final decision is taken by the head of the office.

Special signs testify to the bribes taken by the officials (doctors, teachers). There are the following signs of the person's unusual behavior:

- Age;
- Duration of the period of work at an institution or in a company;
- Purchase of a house, of an apartment, of a summer residence, of a car, etc. at the price not adequate to the level of the wages;
- Debts;

- Financial inquiries;
- Predisposition to gambling;
- The way of life beyond the habitual frameworks;
- Unusual behavior;
- Presence of complaints;
- Vague or criminal past;
- Dishonest or unethical behavior at the office;
- Absence of the division of duties;
- Absence of independent checks;
- Absence of the proper authority;
- Absence of the necessary documents and records;
- The neglect of the existing rules;
- The inadequate system of document circulation, etc.

Elements of the scenario and of the bribe risk LP-model, listed above, are presented by the signs $Z_1, \dots, Z_j, \dots, Z_n$, each of which has several grades. The grades for the j -sign of $Z_{j1}, \dots, Z_{jr}, \dots, Z_{jN_j}$ form GIE. The bribe scenario of the official is described as follows: a bribe can take place if any sign-event takes place or if any two sign-events or all sign-events take place. The scenario of a bribe is given in Fig.4 in the form of a structural graph.

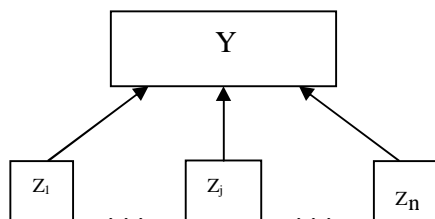


Fig.4. Structural risk model of bribes (\rightarrow -the logical circuit OR)

The construction of the bribe LP-model consists in the calculation of the probabilities P_{jr} , $j=1,2,\dots,n$; $r=1,2,\dots,N_j$ (with which the official takes bribes) on the statistics of the bribe facts established by the courts.

The bribe risk L-function (the bribe risk LP-model) in DNF is [5, 7]

$$(23) \quad Y = Z_1 \vee Z_2 \vee \dots \vee Z_j \vee \dots \vee Z_n.$$

The bribe L-function in the equivalent orthogonal form (ODNF) after the orthogonalization (23) is

$$(24) \quad Y = Z_1 \vee Z_2 \overline{Z_1} \vee Z_3 \overline{Z_2} \overline{Z_1} \vee \dots$$

The bribe risk P-function (model, polynomial) is

$$(25) \quad P = p_1 + p_2 q_1 + p_3 q_1 q_2 + \dots$$

"Arithmetics" in the bribe risk P-model is such that for the final event the bribe probability value is within the limits of [0,1] at any values of the probabilities of initiating events. For every grade-event in GIE (Fig. 1).we use the three probabilities P_{2jr} , P_{1jr} , P_{jr} , introduced before.

The maximum number of different bribes is equal to

$$(26) \quad N_{\max} = N_1 \mathbf{g}^{N_2} \mathbf{g} \cdot \mathbf{g}^{N_j} \mathbf{g} \cdot \mathbf{g}^{N_n},$$

where $N_1, \dots, N_j, \dots, N_n$ are the numbers of the grades in the signs. If the number of signs is equal to $n=20$ and each sign has $N_j=5$ grades, the number of different bribes (conjuncts in a perfect disjunctive normal form - PDNF) equals the astronomical number $N_{\max}=5^{20}$, that explains the difficulties of the struggle with bribes and corruption. (PDNF comprises various conjuncts, each of which comprises all the variables Z_1, Z_2, \dots, Z_n or their denials. The conjuncts are connected by the

logical operation OR).

The bribe risk LP-model (23-25) describes all kinds of bribes and is the most complete and accurate one. In some cases, however, it is not necessary to take into account all possible bribes. For example, it is known from the statistic data that there were bribes when one or two events occurred from Z_1, Z_2, \dots, Z_n . Then, to simplify the model, you should use the bribe risk model for a limited number of bribes.

If we have a logical bribe risk model of four elements

$$(27) \quad Y = Z_1 \vee Z_2 \vee Z_3 \vee Z_4,$$

then for a limited number of bribes, when either one or two events occur, the bribe risk model will be recorded as

$$(28) \quad Y = Z_1 \overline{Z_2} \overline{Z_3} \overline{Z_4} \vee Z_2 \overline{Z_1} \overline{Z_3} \overline{Z_4} \vee Z_3 \overline{Z_1} \overline{Z_2} \overline{Z_4} \vee Z_4 \overline{Z_1} \overline{Z_2} \overline{Z_3} \vee Z_1 Z_2 \overline{Z_3} \overline{Z_4} \\ \vee Z_1 Z_3 \overline{Z_2} \overline{Z_4} \vee Z_1 Z_4 \overline{Z_2} \overline{Z_3} \vee Z_2 Z_3 \overline{Z_1} \overline{Z_4} \vee Z_2 Z_4 \overline{Z_1} \overline{Z_3} \vee Z_3 Z_4 \overline{Z_1} \overline{Z_2}$$

In the bribe risk L-model, all the logical summands are orthogonal in pairs, which allows the bribe risk P-model (P-polynomial) to be written directly:

$$(29) \quad p\{Y\} = p_1 q_2 q_3 q_4 + p_2 q_1 q_3 q_4 + p_3 q_1 q_2 q_4 + p_4 q_1 q_2 q_3 + p_1 p_2 q_3 q_4 + \\ + p_1 p_3 q_2 q_4 + p_1 p_4 q_2 q_3 + p_2 p_3 q_1 q_4 + p_2 p_4 q_1 q_3 + p_3 p_4 q_1 q_2.$$

Example 2. The author did not have any factual data about the bribes, established by the courts on criminal cases. The modeling data were used as the statistical data. From 1000 officials, suspected of bribes, against whom suits were brought, only 300 were condemned, and 700 were considered to be innocent. Thus, the average risk of bribes is equal to $P_{av}=300/1000=0,3$. The suspected officials are described by $n=20$ signs whose total sum is 94 grades.

The identification of the bribe B-model (25) consists in defining the probabilities P_{jr} , $r=1,2,\dots,N_j$; $j=1,2,\dots,n$ for grade-events. The bribe probability for every suspected official is calculated on the optimization step and is compared to the allowable risk P_{ad} . The suspected official is either bad or good. The goal function is formulated as follows: the number of the correctly classified suspected officials should be as great as possible.

Contributions of the grade-events into the accuracy of the bribe LP-model will be considered by us by the example of sign-events (Table 3) of the signs Z_2 and Z_{13} for the optimal identified bribe LP-model ($F_{max}=826$). The grade frequencies for all $P2_{jr}$, for the bad $P2_{0jr}$ and for the good $P2_{1jr}$, the probabilities of the grad-events $P1_{jr}$ and P_j ; for the mistakes of recognition on grades for all E_{jr} , for the bad $E0_{jr}$ and for the good $E1_{jr}$ officials who are under suspicion, are summerized in Table 3;

Table 3. Probabilities and errors of recognition for grade-events of suspected officials

P2jr	P20jr	P21jr	P1jr	Pjr	Ejr	E1jr	E0je
Sign Z_2							
0,014	0,007	0,007	0,010	0,019	0,214	0,429	0,0
0,002	0,001	0,001	0,070	0,014	0,500	1,0	0,0
0,054	0,032	0,022	0,194	0,038	0,278	0,682	0,0
0,017	0,005	0,012	0,159	0,031	0,412	0,5	0,2
0,086	0,038	0,048	0,145	0,028	0,256	0,417	0,053
0,057	0,019	0,038	0,095	0,019	0,228	0,289	0,105
0,224	0,066	0,158	0,067	0,013	0,169	0,196	0,106
0,167	0,056	0,131	0,053	0,010	0,203	0,183	0,250
0,359	0,076	0,283	0,016	0,003	0,114	0,081	0,237
Sign Z_{13}							
0,0190	0,080	0,110	0,283	0,027	0,237	0,345	0,087
0,511	0,142	0,369	0,233	0,021	0,186	0,201	0,148
0,248	0,065	0,183	0,093	0,008	0,113	0,082	0,200
0,028	0,007	0,021	0,346	0,032	0,178	0,238	0,0
0,023	0,006	0,017	0,044	0,004	0,217	0,117	0,5

The contribution of the sign-event into the probability of a bribe by an official is proportional to the probability P_j , $j=1,2,\dots,n$, which equals the probability of the grade-event P_{jr} . The

probabilities P_{jr} of sign grades differ more than 10 times. The grade errors E_{jr} in the classification of bribes differ almost 5 times.

The LP-analysis of the bribe risk model is carried out with the use of (15-18). For each sign j (Table 4): the average values of probabilities PI_{jm} and P_{jm} , were determined and also the decrease of the number of the identified good and bad suspected officials ΔF_j . When this sign was excluded from the bribe model, the bribe LP-model was retrained. The decrease of the number of the suspected officials that could be recognized is determined in relation to the optimal trained bribe model with all signs.

Table 4. The contributions of the signs into the accuracy of the bribe model

Signs, j	Number of grades, Nj	PIjm	Pjm	ΔF_j
1	4	0.272384	0.020226	-64
2	10	0.063346	0.012359	-27
3	5	0.098475	0.009327	-18
4	11	0.090820	0.020927	-26
5	10	0.080377	0.017593	-20
6	5	0.272148	0.022466	-20
7	5	0.206945	0.018549	-6
8	4	0.266619	0.017736	-6
9	4	0.183897	0.014253	-10
10	3	0.318015	0.018295	-10
11	4	0.251871	0.018974	0
12	4	0.247375	0.017166	0
13	5	0.206718	0.018900	-16
14	3	0.235637	0.014733	-2
15	3	0.261648	0.017591	-8
16	4	0.341959	0.021975	-2
17	4	0.289853	0.018739	0
18	2	0.482499	0.017417	0
19	2	0.508613	0.018138	0
20	2	0.750896	0.018326	-2

The maximum contribution into the accuracy of the suspected officials is brought with the sign-events: $Z_1, Z_2, Z_4, Z_5, Z_6, Z_3, Z_{13}$. The zero contribution is brought by the sign-events $Z_{11}, Z_{12}, Z_{17}, Z_{18}, Z_{19}$; excluding sign-events 11,12,17 and 18 reduces the number of the identified suspected only by 4.

The accuracy of the bribe LP-model changes with the change of the number of grades in a sign. The sign Z_2 which in the initial variant had 10 grades was investigated. After retraining the bribe model, the following results were obtained: in the absence of the sign $F_{max}=800$; with two grades $F_{max}=808$; with four grades $F_{max}=812$; with ten grades $F_{max}=824$; with hundred grades (in that case there were seventy empty grades) $F_{max}=828$.

We built a graph of bribe probabilities for 1000 suspected officials before and after sorting according to the value of the probability. Approximately 15 % of the suspected officials had small bribe probabilities and are good, and 15 % of suspected officials had high bribe probabilities and are very bad. It shows that it is necessary to classify the suspected officials according to the bribe probability value into four classes.

5. The bribe risk LP-model on the basis of the analysis of service parameters

Let's estimate the bribe probability using the statistics of the service parameters. These parameters can be, for example, the time it takes the official to solve the problem or it takes the dentist to make a denture (from the very beginning till the end of the process). Such statistics should contain the service precedent number, sufficed for the construction of the discrete or analytical distribution function.

Let us have the statistics (service times) for N clients $Y_i, i=1,2,\dots,N$. If we construct a normal distribution law for the parameter Y_i with the average value and the dispersion, it will lead to an essential decrease of the bribe estimation accuracy.

The service parameter can have either a continuous or discrete values. In both cases, with the purpose of increasing the bribe model adequacy and using the apparatus of LP-calculation, we shall build the discrete distribution on the chosen intervals of splitting the parameter values. We give the grade number for each interval. The grades make group of incompatible events (GIE). The probabilities of grade-events are determined by the formula

$$(30) \quad P_j = N_j / N,$$

where: N_j is the number of the parameter values in the statistics with the given grades; N is the number of the parameter values in the statistics.

The service parameter has the average value Y_m and the allowable value Y_{ad} (Fig. 5). The probability $P\{Y < Y_{ad}\}$ will be named by us the bribe risk. The scenario of the bribe is formulated as follows: if the service parameter is greater (smaller) than the allowable value, then we suspect that there must have been a bribe.

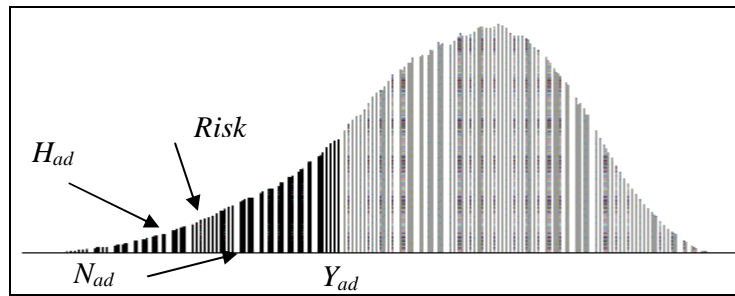


Fig. 5. Discrete distribution of the service parameter

Thus, for the service parameter at the given *Risk* we can compute the following: the admitted value Y_{ad} , the number of the values of the parameter in the “tail” of the distribution N_{ad} , the entropy of the probabilities of the parameter in “tail” of the distribution H_{ad} .

In numerous publications in the field of *VaR* (*Value-at-Risk*) theory, the authors investigated tails of distribution. For these purpose various distributions and conditional probabilities are suggested, which have no rigorous justification. In the bribe LP-theory it is not to be done because we use any distribution laws given by a discrete line.

Example 3. $N=700$ clients have been served. The parameter Y_1 determines the duration of the service by days and has $N_1=30$ days-grades. The probabilities P_{1r} are calculated, $r=1,2,\dots,30$ by (28). The admitted parameter value is $Y_{1ad}=10$ and the risk value is $Risk_1=0,2$. The suspicion of the bribe is caused if $Y_1 < Y_{1ad}$.

Let there be one more the service parameter Y_2 having $N_2=20$ grades, the admitted parameter value Y_{2ad} and the risk $Risk_2$. The logical variables correspond to the service parameters. The logical variables can be dependent, but is not initially, for they are contained in the certain logic formula which determines the dependence between them.

For the case of two service parameters Y_1 and Y_2 , we have $N=N_1*N_2=30*20=600$ combinations of service. The L-functions for two different service combinations Y_1Y_2 are orthogonal (the product of the logic functions of different combinations is equal to zero) as these combinations contain different grades for Y_1 and Y_2 , belonging to GIE.

The property of orthogonality of different service combinations allows us to pass from the L-functions to the algebraic expressions for probabilities, that is the L-variables are to be replaced by the probabilities and the signs “or” are to be replaced by “plus”.

It is easy to calculate the number of combinations satisfying the bribe condition

$$(31) \quad P\{(Y_1 \leq Y_{1ad}) \vee (Y_2 \leq Y_{2ad})\},$$

and calculate the bribe probability for this condition.

6. Conclusion

Problems of identifying the bribe LP-models, the estimation and the analysis of bribe probabilities on the basis of the risk LP-theory with groups of incompatible events, as follows from expression (19, 26), have an extremely high computing complexity and can be solved only by means of modern computers and special *Software*. A complex of *Software* for the solution of all those problems of training, modeling and of the analysis of bribes has been elaborated. You may get the detailed information on these *Software* in [5,7,9], on www.inorisklab.com, www.ipme.ru/ipme/labs/iisad/sapr.htm, и по E-mail: risk@sapr.ipme.ru.

Basic results of the present work are the following:

1. It is offered to use the risk LP-theory with groups of incompatible events for the development of the bribe risk LP-models with the purpose of revealing, estimating and analyzing their probabilities on the evidence of the statistical data.
2. The construction of the LB- model comprises the following:
 - The presentation of the risk L-model of the bribe in PDNF in order to estimate the number of various possible combinations of bribes and the computing complexity of algorithms;
 - If the computing complexity is great, it should record the bribe risk L-model in DNF, using the scenario of the bribes in the form of remarks, or with help of the graph, or in the form of the shortest ways; or for the limited set of bribes.
 - The transformation of the risk L-model of a bribe from DNF into ODNF;
 - Recording the risk B-model of the bribe according to ODNF;
 - Identifying the risk B-model of the bribe according to the statistical data taking into account of GIE;
 - Analysis of the risk B-model of the bribe with the calculation of the contribution of the signs and grades into the possibility of a bribe, the average probability of bribes and the exactness of the risk LB-model.
3. Scenarios and the risk LP-models are described for bribes:
 - at the institutions according to the results of their functioning,
 - of the officials on the basis of the descriptions of their behavior,
 - at the institutions and of the officials on the basis of the analysis of service parameters.
4. Examples are given of the estimation and analysis of the probabilities of bribes on the basis of identifying the bribe risk LP-model on the statistical data.
5. The developed LP-models of bribes can be used both individually and all together.
6. Software has been developed for training the risk LP-model, and for the estimation and analysis of the probabilities of bribes. It is intended for the department “Economic crimes” of towns and services of safety and control of banks and companies with the purpose of revealing bribes on statistical data.

Bibliography

1. Albrecht W., Wernz G., Williams T., Fraud. Bringing Light to the Dark Side of Business. Transl. from English. Saint Petersburg: Piter, 1995, 396 p.
2. Satarov G. A. Anticorruption politics. The manual, M.: RA SPAS, 2004, 368 p.
3. Eliseeva I. I. General statistics theory: The text book. – M.: Finance and statistics, 2004. – 656 p.
4. Heckman James J., Leamer Edward. Handbook of Econometrics, v.5, 2002.
5. Solozhentsev E. D. Scenario logic and probabilistic management of risk in business and engineering. Second Edition. Saint Petersburg, Business - Press, 2006, 560 p.
6. Ryabinin I. A., Reliability and safety of structure-complex systems. Saint Petersburg: Politechnika, 2000, 248 p.
7. Solojntsev E. D. Scenario Logic and Probabilistic Management of Risk in Business and Engineering. Springer: 2004, 391 p.

8. Solozhentsev E.D., Stepanova N.V., Karasev V.V. Transparency of methods for assessment of credit risks and ratings. Saint Petersburg: Publishing house St. Petersburg University Press, 2005, 196 p.
9. Solojentsev E. D., Karassev V. V., Solojentsev V.E. Logic and probabilistic models of risk in banks, business and quality. Saint Petersburg: Nauka, 1999, 120 p.